141 Reasoning About Programs 2017

1a We use induction over list structure to prove the assertion

Base case

To show Vys a revz I ys rev EI tt ys

Take an arbitary ys Ea

rev2 CI ys ys By definition of rer2 rev 1 ttys Itt ys CBy definition of rev

ys By Lemma B

Inductive case

To show i Ves a Uys a rev2 Xs ys rev xDttys Vxiarees xix ys

rev xias ttys

Take an arbitany Xs Eat and ys EI xia as well

Inductive hypothesis Assume that rev2 as ys Crow xs ys holds

her Lxixs ys neck Xs xiys By definition of nerd

rev xsitcx.gs By Inductive hypothesis

rev xsitt 43 ttys By Lemma A

rev x xs ttys By definition of red

Hence we have proved the assertion

2b i Inductive principle VnK2 K2 k3iZ Pcn n Kl K2 k3 Kl Attn cut kl k2

k3 riz ntcn rr hcn.cntl klxk3 kl k2nPCn cnttl kl k3 ki K2 r E Pcn Cat Kl k2 k3DL Unont hi K2k3 r 7 h h cut biR2 k3 r 7 us Pcn ont ki K2 k3 r

Ii Base case To show that Vn ki ka k3 2 nZ3AklfGMk2 fCn 1 A

g k3 fCn 2 7 klatch asI Take arbitary n ki K2k3 2 and we start the proof

my proo

Inductive case

Take arbitary n ont kl k2 k3 r I Assume that Vn ontki R2 k3 ri 2 ntcntnhl.in Catti kl k3 ki kz r

Inductive hypothesis in 31 kl k3 f cut141kl f cutAk2 float 1 r fCn

we need to show that Vncut hi k2k3 ri 2 hcncatKl k2k3 v

nzzrkkfccn jhkz fcent ijhk3 f.cat 2 r flea

we begin the proof with the inductive hypothesis

iii n z 0 or VniIN

Iv We prove termination when n 0 I 2 base case and then

use mathematical induction to prove that h will terminate if NZcht thus g will terminate for any n 2 as well V We first prove that for n o I z f n g n holds

Then follow from C in part ii that for n 3 hln 2,30 Zo lo

r fn Hence g n f n for n 73 as well Hence UnilN f n g n

2 a i factors l I ii factors4 a 2,21

iii factors 7 i 7 o o IV factors lo EZ J O o o

b i when input i I

iis On line 5 the creation of the array Is requires its size to

be positive

iii PRE nzo Notice that new inteo2 is allowed in Java

Iv Not all elements in the final array are factors of n There

might be 0 when for instance calling factorsClo V IposEI vlength VyEEO pos rcydtOAI m IN.mxr.ly n1

This captures the situation when neo as well Since has length0

this postcondition will be vacuously true

C We will prove that R Q Note Q is modified as in b b

Given 1 TITTYten rwrapperEk n

2 AE arE Aten\_searchCa07hIposCEo r.lengthf.VyEa.pro ryJ not sure aboutthis Implied from factor postondition andJava's array initialization to 0

3 ret length ten

4 OEontE ree length Loop invariant

J ont Z ret length termination of whiteloop 6 net6 cut TnACO art Loop invariant

7 runapper Tn ret E

To show 8 IposC o nlength VyEco pos rly 1 01 7malN.mxr.ly n

when wrapperlength O i 8 is vacuously true

When wrapperlength L i rEOI n From 1

we can let m l and then m rEoI n

Thus 8 holds in this case

when wrapperlength l i

Fron l 2 and definition of search function we know that

9 ten pos From 9 and 3 we know w retlength pos

From 4 andJ we know 11 cut retelength

From iosand11 we know 14 Cutepros From 6 12 and 7 we know 14 runapperE Tereteposterat Thionate rwameLi For any Repos we can let m z and thus m rEk7 n From 14

87 holds in this case

d i INV d LE Curren 12 EcandEn n OEpose r length.A Curr TIMEfsEk2 n

ii VAR Curr cand ceach time either curr decreases or

cand increases

e i Rnew E RA Vie6 vlength rei I AVa.beN rLiI axb a IVb D

Ii It comes from the fact that the function factors

produces the prime factorization of n Suppose there exists

a composite number in fs its factors must be already in

fs as well but then curr to cand 0 would be

false since Curr has been updated on line 5

Hence this composite number won't be added to Is